NMTC 2016

JUNIOR IX & X

ANSWER KEYS

WITH SOLUTION

1. The sum of the values x, y that satisfy the equations

$$(x+y)2^{y-x} = 1, \quad (x+y)^{x-y} = 2$$

simultaneously is

A. 2 B. $\frac{3}{2}$ C. $\frac{5}{2}$ D. $\frac{7}{2}$

ANS: [1] (C) $\frac{5}{2}$

$(x+y)\,2^{y-x} = 1 \Rightarrow (x+y) = \frac{1}{2^{y-x}} \Rightarrow (x+y) = 2^{x-y}$ —— ①

&, $(x+y)^{x-y} = 2 \Rightarrow (x+y) = 2^{\frac{1}{x-y}}$ —— ⑪

From $\frac{①}{⑪} \Rightarrow \frac{2^{x-y}}{2^{\frac{1}{x-y}}} = 1 \Rightarrow 2^{x-y} = 2^{\frac{1}{x-y}}$

$\Rightarrow x-y = \frac{1}{x-y}$

$\Rightarrow (x-y)^2 - 1 = 0$

$\Rightarrow \cancel{(x-y)(x-y}$

$\Rightarrow (x-y+1)(x-y-1) = 0$

$\Rightarrow \boxed{x-y = -1 \text{ or } 1}$

Thus,

$x+y = 2^{x-y} = 2$ if $x-y = 1$

$x+y = \frac{1}{2}$ if $x-y = -1$

Now, on solving

$x+y = 2$ and $x-y = 1$

$x+y = \frac{1}{2}$ and $x-y = -1$

We get

$x = \frac{3}{2}$ or $\frac{-1}{4}$, $y = \frac{1}{2}$ or $\frac{3}{4}$

Sum of all values of x & y = $\boxed{\frac{5}{2}}$

2. If

$$\left(2-\frac{a}{4}-\frac{4}{a}\right)\left\{(a-4)\sqrt[3]{(a-4)^{-3}}-\frac{(a^2-16)^{-\frac{1}{2}}(a-4)^{-\frac{1}{2}}}{(a+4)^{-\frac{3}{2}}}\right\}\left(\frac{a+4}{a-4}\right)=2016$$

the value of a is

A. $\frac{4}{1007}$ B. $\frac{3}{2016}$ C. $\frac{4}{2017}$ D. none of these

ANS: [2] (A) $\frac{4}{1007}$

$$\left(2-\frac{a}{4}-\frac{4}{a}\right)\left\{(a-4)\sqrt[3]{(a-4)^{-3}}-\frac{(a^2-16)^{-\frac{1}{2}}(a-4)^{-\frac{1}{2}}}{(a+4)^{-\frac{3}{2}}}\right\}\left(\frac{a+4}{a-4}\right)=2016$$

$$\Rightarrow \left(\frac{8a-a^2-16}{4a}\right)\left\{\frac{a-4}{a-4}-\frac{(a-4)^{-\frac{1}{2}}(a+4)^{-\frac{1}{2}}(a-4)^{-\frac{1}{2}}}{(a+4)^{-\frac{3}{2}}}\right\}\left(\frac{a+4}{a-4}\right)=2016$$

$$\Rightarrow \frac{-(a-4)^2}{4a}\left\{1-\frac{(a+4)}{(a-4)}\right\}\left(\frac{a+4}{a-4}\right)=2016$$

$$\Rightarrow \frac{-\cancel{(a-4)^2}}{4a}\times\frac{\cancel{a}-4-\cancel{a}-4}{\cancel{(a-4)}}\times\frac{a+4}{\cancel{(a-4)}}=2016$$

$$\Rightarrow \frac{\cancel{8}\,2\,(a+4)}{\cancel{4}a}=\cancel{2016}\;1008$$

$$\Rightarrow a+4=1008a$$

$$\Rightarrow \boxed{a=\frac{4}{1007}}$$

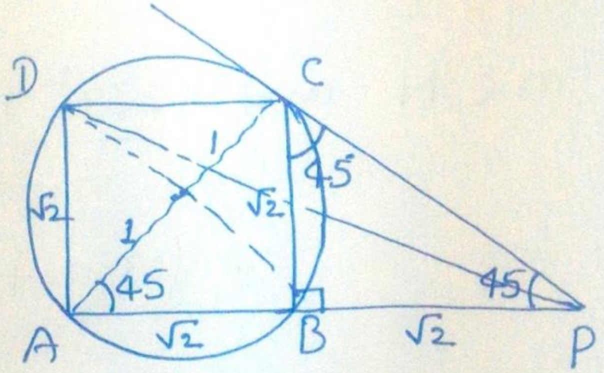
3. $ABCD$ is a square inscribed in a circle of radius 1 unit. The tangent to the circle at C meets AB produced at P. The length of PD is

A. 2 B. 3 C. $\sqrt{13}$ D. $\sqrt{10}$

ANS: [3] (D) $\sqrt{10}$

$\because$ Diagonal of square $= 2$

$\Rightarrow$ side $= \sqrt{2}$



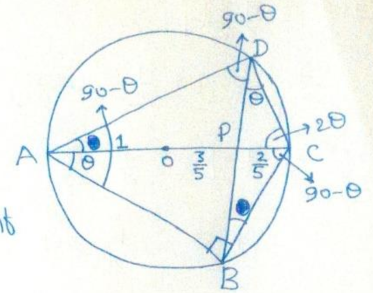
Thus,

$$PD = \sqrt{AD^2 + AP^2} = \sqrt{2 + 8} = \boxed{\sqrt{10}}$$

4. Quadrilateral $ABCD$ is inscribed in a circle with radius 1 unit. AC is the diameter of the circle and $BD = AB$. The diagonals cut at P. If $PC = \frac{2}{5}$ then the length of CD is equal to

A. $\frac{2}{3}$ B. $\frac{2}{7}$ C. $\frac{1}{8}$ D. $\frac{3}{4}$

ANS: [4]



Let $\angle BAC = \theta$

$\angle BCA = 90 - \theta$

By Angle in a same segment

$\angle BDC = \theta$

$\angle BDA = 90 - \theta$

$\because AB = BD \Rightarrow \angle BAD = \angle BDA = 90 - \theta$

$\Rightarrow \angle CAD = 90 - \theta - \theta = 90 - 2\theta$

By Angle sum property of $\triangle$

$\angle APD = 3\theta$ and $\angle ACD = 2\theta$

In $\triangle PDC$, $\frac{PD}{\sin 2\theta} = \frac{PC}{\sin\theta}$ —— (1)

$\triangle PAD$, $\frac{PD}{\cos 2\theta} = \frac{AP}{\cos\theta}$ —— (11)

From $\frac{(1)}{(11)} \Rightarrow \cot 2\theta = \frac{PC}{AP}\cot\theta$

$\because \frac{PC}{AP} = \frac{1}{4} \Rightarrow \tan\theta = \frac{1}{4}\tan 2\theta$

$\Rightarrow \tan^2\theta = \frac{1}{\sqrt{2}}$

In $\triangle ADC$, $DC = AC\cos 2\theta = AC\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \boxed{\frac{2}{3}}$

5. The number of natural numbers n for which the expression $\dfrac{23n^2+18n+4}{n}$ is also a natural number is

A. 3 B. 2 C. 1 D. 0

ANS: [5] (A) 3

$\because n \in N$ and $\dfrac{23n^2+18n+4}{n} \in N$

$\Rightarrow \left(23n+18+\dfrac{4}{n}\right) \in N$

Thus, n must be 1, 2, 4.

$\Rightarrow$ Number of natural numbers n is $\boxed{3}$

6. The cost price of 16 oranges is equal to the selling price of 12 oranges. Then there is a

A. 40% profit B. 20% loss C. $33\frac{1}{3}$ % profit D. $23\frac{1}{3}$ % profit

ANS: [6] (c) $33\frac{1}{3}$ % profit

Let the CP of 16 oranges = SP of 12 oranges = x

$\Rightarrow$ CP = $\frac{x}{16}$ and SP = $\frac{x}{12}$

$\Rightarrow$ profit = $\frac{x}{12} - \frac{x}{16} = \frac{x}{48}$

Thus, the profit % = $\dfrac{\frac{x}{48} \times 100}{\frac{x}{16}} = \boxed{33\frac{1}{2}\,\%}$

7. The number of positive integer pairs (a, b) such that $ab - 24 = 2b$ is

A. 6 B. 7 C. 8 D. 9

ANS: [7] (c) 8.

$ab - 24 = 2b \Rightarrow ab - 2b = 24$

$\Rightarrow b(a-2) = 24$

$\Rightarrow b = \dfrac{24}{a-2}$

$\because$ a and b are positive integers.

$\Rightarrow a-2 = 1, 2, 3, 4, 6, 8, 12, 24$

$\Rightarrow a = 3, 4, 5, 6, 8, 10, 14, 26$

$b = 24, 12, 8, 6, 4, 3, 2, 1$

Thus, the number of pairs $(a, b) = \boxed{8}$

8. $A = (2+1)(2^2+1)(2^4+1)\cdots(2^{2016}+1)$. The value of $(A+1)^{1/2016}$ is

A. 4 B. 2016 C. 2^{4032} D. 2

ANS: [8] (A) 4

If $A = (2^1+1) \Rightarrow (A+1) = 2+2 = 2^2$

If $A = (2+1)(2^2+1) \Rightarrow (A+1) = 15+1 = 16 = 2^4$

If $A = (2+1)(2^2+1)(2^4+1) \Rightarrow (A+1) = 255+1 = 256 = 2^8$

similarly

If $A = (2+1)(2^2+1)(2^4+1)\ldots(2^{2016}+1)$

$\Rightarrow (A+1) = 2^{2\times 2016}$

$\Rightarrow (A+1)^{\frac{1}{2016}} = 2^2 = \boxed{4}$

9. The sum of two numbers a, b where $a < b$ is 1215 and their H.C.F is 81. The number of pairs of such pairs (a, b) is

A. 1 B. 2 C. 3 D. 4

ANS: [9] (D) 4.

$\because$ HCF $(a, b) = 81$

Let $a = 81x$, $b = 81y$,

where $x < y$ and HCF$(x, y) = 1$

$\because$ $a + b = 1215$

$\Rightarrow 81(x + y) = 1215$

$\Rightarrow x + y = 15$

Thus, the possible values of x and y such that HCF$(x, y) = 1$ and $x + y = 15$ are

$(x, y) \equiv (1, 14), (2, 13), (4, 11), (7, 8)$

Thus, the number of pairs (a, b) is $\boxed{4}$

10. The first Republic Day of India was celebrated on 26th January 1950. What was the day of the week on that date?

A. Tuesday B. Wednesday C. Thursday D. Friday

ANS: [10] (C) Thursday

26 jan 1950 = 1949 years + Period from 1 jan 1950 to 26 jan 1950

No. of odd days in 1600 years = 0 odd days

300 —— = 1 odd day.

49 years = 12 Leap years + 37 Non-leap years

= (12 × 2 + 37 × 1) odd days

= 61 odd days

= 8 weeks + 5 odd days

In period from 1 jan 1950 to 26 jan 1950

= 3 weeks + 5 odd days

Thus total no. of odd days

= 0 + 1 + 5 + 5 = 11 odd days

= 1 week + 4 odd days

= 4 odd days

Therefore, Day on 26 jan 1950 = $\boxed{\text{Thursday}}$

11. The 12 numbers $a_1, a_2, \ldots, a_{12}$ are in arithmetical progression. The sum of all these numbers is 354. Let $P = a_2 + a_4 + \cdots + a_{12}$ and $Q = a_1 + a_3 + \cdots + a_{11}$. If the ratio $P : Q$ is $32 : 27$, the common ratio of the progression is

A. 2 B. 3 C. 4 D. 5

ANS: [11] (D) 5.

$a_1 + a_2 + a_3 + \cdots + a_{12} = 354$

$\Rightarrow \frac{12}{2}[2a_1 + 11d] = 354$

$\Rightarrow 2a_1 + 11d = 59$ —— (i)

$P = a_2 + a_4 + \cdots + a_{12} = \frac{6}{2}[2a_2 + 5(2d)]$

$= 6(a_2 + 5d)$

$Q = a_1 + a_3 + \cdots + a_{11} = \frac{6}{2}[2a_1 + 5(2d)]$

$= 6(a_1 + 5d)$

$\because \frac{P}{Q} = \frac{a_2 + 5d}{a_1 + 5d} = \frac{32}{27} \Rightarrow$

$\Rightarrow \frac{a_1 + 6d}{a_1 + 5d} = \frac{32}{27}$ $(\because a_2 = a_1 + d)$

$\Rightarrow \frac{2a_1 + 12d}{2a_1 + 10d} = \frac{32}{27}$

$\Rightarrow \frac{59 - 11d + 12d}{59 - 11d + 10d} = \frac{32}{27}$ [From (i)]

$\Rightarrow \frac{59 + d}{59 - d} = \frac{32}{27}$

$\Rightarrow \boxed{d = 5}$

12. A shopkeeper marks the prices of his goods at 20% higher than the original price. There is an increase in demand of the goods, and he further increases the price by 20%. The total profit % is

A. 40 B. 38 C. 42 D. 44

ANS: [12] (D) 44.

Let $CP = x$

$$SP = x\left(1+\frac{20}{100}\right)\left(1+\frac{20}{100}\right) = \frac{36x}{25}$$

$$\text{Profit} = \frac{36x}{25} - x = \frac{11x}{25}.$$

Thus, profit % $= \frac{11x}{25} \times 100 \times \frac{1}{x}$

$$= \boxed{44\%}$$

13. A circle passes through the vertices A and D and touches the side BC of a square. The side of the square is 2 cm. The radius of the circle (in cm) is

A. $\frac{5}{4}$ B. $\frac{4}{5}$ C. 1 D. $\frac{5}{2}$

ANS: [13] (A) $\frac{5}{4}$

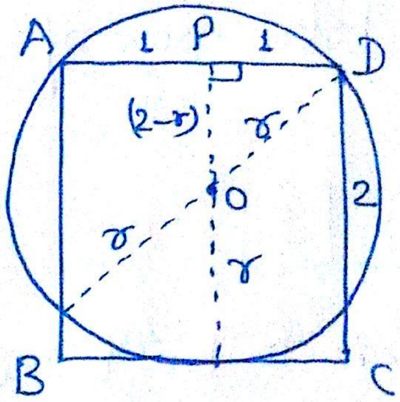
In $\triangle OPD$,

$$(2-r)^2 + 1 = r^2$$

$$\Rightarrow 4 + r^2 - 4r + 1 = r^2$$

$$\Rightarrow 4r = 5$$

$$\Rightarrow \boxed{r = \frac{5}{4}}$$



14. There are four balls – one green, one red, one blue and one yellow and there are four boxes – one green, one red, one blue and one yellow. A child playing with the balls decides to put the balls in the boxes, one ball in each box. The number of ways in which the child can put the balls in the boxes such that no ball is in a box of its own color is

A. 12 B. 9 C. 24 D. 6

ANS: [14]

According to Dearrangment formula, we have

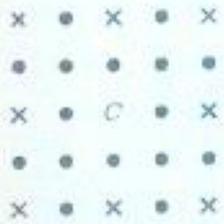
$n = \lfloor 4 = 24$

Required probability

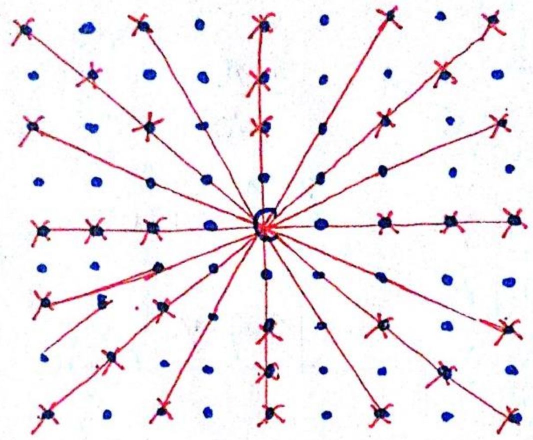
$= (24)\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = \boxed{9}$

15. The 5×5 array of dots represents trees in an orchard. If you were standing at the central spot marked C, you would not be able to see 8 of the 24 trees (shown as X). If you were standing at the center of a 9×9 array of trees, how many of the 80 trees would be hidden?

A. 40 B. 32 C. 36 D. 44



ANS: [15] (B) 32



Thus, total no of hidden trees = $\boxed{32}$

16. a and b are positive integers such that $a^2 + 2b = b^2 + 2a + 5$. The value of b is ———

ANS: [16] $\boxed{3}$.

$a^2 + 2b = b^2 + 2a + 5$

$\Rightarrow (a^2 - b^2) - 2(a-b) = 5$

$\Rightarrow (a-b)(a+b-2) = 5 \times 1$

$\Rightarrow a+b = 7$

$a-b = 1$

$\Rightarrow a = 4, \boxed{b = 3}$

17. After full simplification, the value of the product

$$\left(\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2+\sqrt{2+\sqrt{3}}}\right)\left(\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\right)\left(\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}\right)$$

is ______

ANS: [17] $\boxed{1}$

$$\left(\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2+\sqrt{2+\sqrt{3}}}\right)\left(\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\right)\left(\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}\right)$$

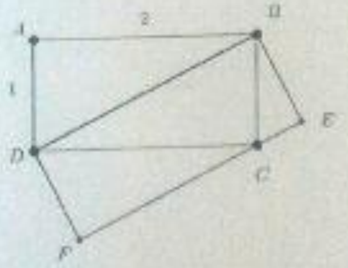
$$=\left(\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2+\sqrt{2+\sqrt{3}}}\right)\left\{\sqrt{4-2-\sqrt{2+\sqrt{3}}}\right\}$$

$$=\left(\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2+\sqrt{2+\sqrt{3}}}\right)\left(\sqrt{2-\sqrt{2+\sqrt{3}}}\right)$$

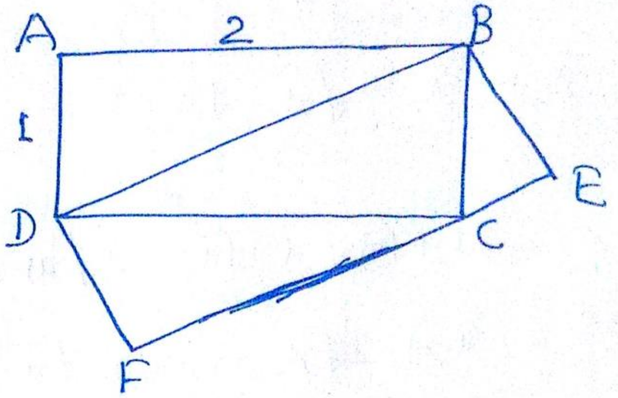
$$=\left(\sqrt{2+\sqrt{3}}\right)\left(\sqrt{4-2-\sqrt{3}}\right)$$

$$=\left(\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2-\sqrt{3}}\right)=\left(\sqrt{4-3}\right)=\sqrt{1}=\boxed{1}$$

18. $ABCD$ is a rectangle with $AD = 1$ and $AB = 2$. $DFEB$ is also a rectangle. The area of $DFEB$ is ______



ANS: [18] $\boxed{2}$



ar □DFEB

$= 2(\text{ar}\,\triangle DCB)$

$= 2\left(\frac{1}{2} \times \text{ar}\,□ABCD\right)$

$= \text{ar}\,□ABCD$

$= 2 \times 1 = \boxed{2}$

19. The two digit number whose units digit exceeds the tens digit by 2 and such that the product of the number and the sum of its digits is 144 is ———

ANS: [19] $\boxed{24}$

Ten's Digit $= x$, Unit's Digit $= (x+2)$

$(10x + x + 2)(x + x + 2) = 144$

$\Rightarrow (x+1)(11x+2) = 72$

$\Rightarrow 11x^2 + 13x - 70 = 0$

$\Rightarrow (11x+35)(x-2) = 0$

$\Rightarrow x = 2$

Thus, Required No $= \boxed{24}$

20. If $x = \frac{p}{q}$ where p, q are integers having no common divisors other than 1, satisfies

$$\sqrt{x+\sqrt{x}} - \sqrt{x-\sqrt{x}} = \frac{3}{2}\sqrt{\frac{x}{x+\sqrt{x}}}$$

then x is ——

ANS: [20] $\boxed{\frac{25}{16}}$

$$\sqrt{x+\sqrt{x}} - \sqrt{x-\sqrt{x}} = \frac{3}{2}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}$$

$$\Rightarrow 2\sqrt{x+\sqrt{x}}\sqrt{x+\sqrt{x}} - 2\sqrt{x+\sqrt{x}}\sqrt{x-\sqrt{x}} = 3\sqrt{x}$$

$$\Rightarrow 2x + 2\sqrt{x} - 2\sqrt{x}\sqrt{x-1} = 3\sqrt{x}$$

$$\Rightarrow 2\sqrt{x} + 2 - 2\sqrt{x-1} = 3$$

$$\Rightarrow 2\sqrt{x} - 2\sqrt{x-1} = 1$$

$$\Rightarrow 2\sqrt{x} = 2\sqrt{x-1} + 1$$

$$\Rightarrow 4x = 4(x-1) + 1 + 4\sqrt{x-1}$$

$$\Rightarrow \sqrt{x-1} = \frac{3}{4} \Rightarrow \boxed{x = \frac{25}{16}}$$

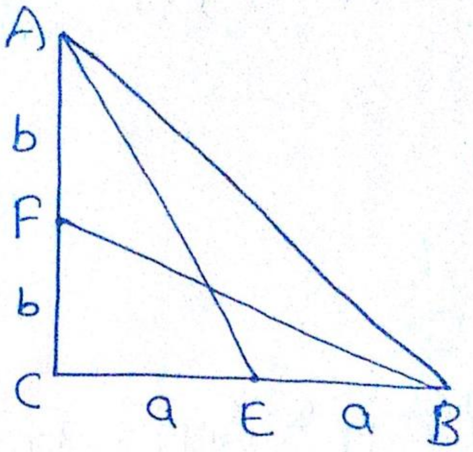
21. AE and BF are medians drawn to the legs of a right angled triangle ABC. The numerical value of $\dfrac{AE^2 + BF^2}{AB^2}$ is ____

ANS: [21] $\boxed{\dfrac{5}{4}}$

$$\frac{AE^2+BF^2}{AB^2} = \frac{4b^2+a^2+a^2+4a^2}{4a^2+4b^2}$$

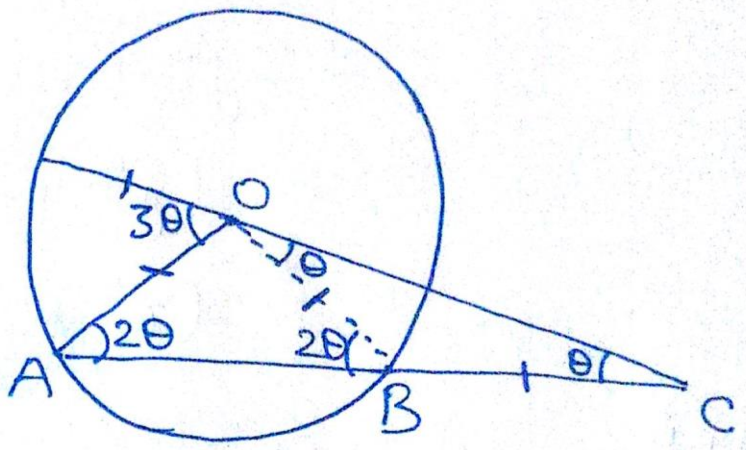
$$= \frac{5(a^2+b^2)}{4(a^2+b^2)}$$

$$= \boxed{\frac{5}{4}}$$



22. AB is a chord of a circle with center O. AB is produced to C such that $BC = OA$. CO is produced to E. The value of $\frac{\angle AOE}{\angle ACE}$ is ———

ANS: [22] $\boxed{3}$



Thus,

$$\frac{\angle AOE}{\angle ACE} = \frac{3\theta}{\theta} = \boxed{3}$$

23. The number of two digit numbers that are less than the sum of the squares of their digits by 11 and exceed twice the product of their digits by 5 is ———

ANS: [23] $\boxed{2}$

$$10A + B = A^2 + B^2 - 11 = 2AB + 5$$

From Last Two $\Rightarrow (A-B)^2 = 16 \Rightarrow A - B = \pm 4$

From First & Last $\Rightarrow 10A + B - 2AB - 5 = 0$

$\Rightarrow (2A-1)(5-B) = 0$

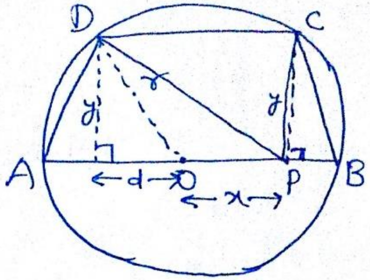
$\Rightarrow \boxed{B = 5} \Rightarrow \boxed{A = 9, \text{ or } 1}$

$\boxed{95 \text{ or } 15}$

No. of two digit No $= \boxed{2}$

24. AB is a diameter of circle and CD is a parallel chord. P is any point in AB. The numerical value of $\dfrac{PC^2 + PD^2}{PA^2 + PB^2}$ is ———

ANS: [24] $\boxed{1}$



$\because PA^2 + PB^2 = (r+x)^2 + (r-x)^2$

$= 2(r^2 + x^2)$

$PC^2 + PD^2 = y^2 + (d-x)^2 + y^2 + (d+x)^2$

$= 2y^2 + 2(d^2 + x^2)$

$= 2(y^2 + d^2) + 2x^2$

$= 2(r^2 + x^2)$

Thus, $\dfrac{PC^2 + PD^2}{PA^2 + PB^2} = \boxed{1}$

25. In the sequence $1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, \ldots$ the 2016th term is 2^n. Then $n =$ ______

ANS: [25] $\boxed{10}$

First 1 is at term 1

— 2 — 2

— 4 — 4

— 8 — 8

⋮

— 1024 — 1024

— 2048 — 2048

$\Rightarrow$ 2016th term $= 1024 = 2^{10} \Rightarrow \boxed{n = 10}$

26. Each root of the equation $ax^2 + bx + c = 0$ is decreased by 1. The quadratic equation with these roots is $x^2 + 4x + 1 = 0$. The numerical value of $b + c$ is ———

ANS: [26] $\boxed{0}$

Let α and β are roots of $ax^2+bx+c=0$

$\Rightarrow \alpha+\beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

Then,

$(\alpha-1)$ and $(\beta-1)$ are roots of $x^2+4x+1=0$

$\alpha+\beta-2 = -4 \Rightarrow \alpha+\beta = -2$

$\Rightarrow \frac{-b}{a} = -2$

$\Rightarrow \boxed{b = 2a}$

$(\alpha-1)(\beta-1) = 1$

$\Rightarrow \alpha\beta - (\alpha+\beta) + 1 = 1$

$\Rightarrow \frac{c}{a} - (-2) = 0$

$\Rightarrow \boxed{c = -2a}$

Thus, $b+c = 2a - 2a = \boxed{0}$

27. The number of integers n such that $\frac{n+2}{n^2+1} > \frac{1}{2}$ is ——

ANS: [27] $\boxed{3}$

$$\frac{n+2}{n^2+2} > \frac{1}{2} \Rightarrow (n-3)(n+1) < 0$$

$\Rightarrow$ No. of integeral values $= \boxed{3}$

28. P_1 and P_2 are two regular polygons. The number of sides of P_1 and P_2 respectively are in the ratio $3:2$ and the respective interior angles are in the ratio $10:9$. Then the sum of the number of sides of P_1 and P_2 is ———

ANS: [28] $\boxed{20}$

$$\frac{P_1}{P_2} = \frac{3}{2} \Rightarrow P_1 = 3x \text{ and } P_2 = 2x$$

$$\frac{(P_1 - 2) \times \cancel{180}}{P_1} \times \frac{P_2}{(P_2 - 2) \times \cancel{180}} = \frac{10}{9}$$

$$\Rightarrow \frac{2(3x-2)}{3(2x-2)} = \frac{10}{9}$$

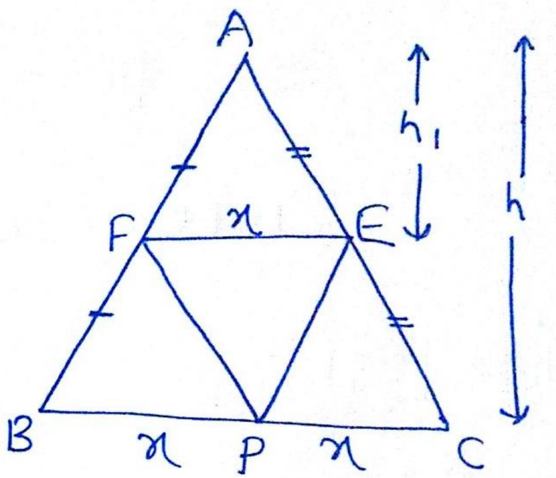
$$\Rightarrow x = 4$$

$$\Rightarrow P_1 = 12 \text{ and } P_2 = 8$$

Thus, $P_1 + P_2 = \boxed{20}$

29. In triangle ABC, F and E are the mid points of AB and AC respectively. P is any point on the side BC. The ratio $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta FPE}$ is ———

ANS: [29] $\boxed{4}$



By Mid point Theorem

$EF = \frac{1}{2} BC = x$

$\because \triangle AEF \sim \triangle ACB$

$\Rightarrow \quad \frac{FE}{BC} = \frac{1}{2} = \frac{h_1}{h}$

Thus, $\frac{ar \triangle ABC}{ar \triangle FPE} = \frac{\frac{1}{2} \times BC \times h}{\frac{1}{2} \times EF \times (h-h_1)} = \boxed{\frac{4}{1}}$

30. x, y, z are distinct real numbers such that

$$x+\frac{1}{y}=y+\frac{1}{z}=z+\frac{1}{x}$$

The value of $x^2y^2z^2$ is ———

ANS: $\boxed{30}$ $\boxed{1}$

$$x+\frac{1}{y}=y+\frac{1}{z}=z+\frac{1}{x}$$

$$\Rightarrow \quad x-y=\frac{1}{z}-\frac{1}{y}=\frac{y-z}{yz} \quad \text{——— (I)}$$

$$\Rightarrow \quad y-z=\frac{1}{x}-\frac{1}{z}=\frac{z-x}{xz} \quad \text{——— (II)}$$

$$\Rightarrow \quad z-x=\frac{1}{y}-\frac{1}{x}=\frac{x-y}{xz} \quad \text{——— (III)}$$

Thus, (I) × (II) × (III)

$$\Rightarrow (x-y)(y-z)(z-x)=\frac{(x-y)(y-z)(z-x)}{(xyz)^2}$$

$$\Rightarrow \quad \boxed{x^2y^2z^2=1}$$